Control Systems - Introduction

A control system is a system, which provides the desired response by controlling the output. The following figure shows the simple block diagram of a control system.

Here, the control system is represented by a single block. Since, the output is controlled by varying input, the control system got this name. We will vary this input with some mechanism. In the next section on open loop and closed loop control systems, we will study in detail about the blocks inside the control system and how to vary this input in order to get the desired response.

Examples – Traffic lights control system, washing machine

Traffic lights control system is an example of control system. Here, a sequence of input signal is applied to this control system and the output is one of the three lights that will be on for some duration of time. During this time, the other two lights will be off. Based on the traffic study at a particular junction, the on and off times of the lights can be determined. Accordingly, the input signal controls the output. So, the traffic lights control system operates on time basis.

Classification of Control Systems

Based on some parameters, we can classify the control systems into the following ways.

Continuous time and Discrete-time Control Systems

- Control Systems can be classified as continuous time control systems and discrete time control systems based on the **type of the signal** used.
- In **continuous time** control systems, all the signals are continuous in time. But, in **discrete time** control systems, there exists one or more discrete time signals.

SISO and MIMO Control Systems

- Control Systems can be classified as SISO control systems and MIMO control systems based on the **number of inputs and outputs** present.

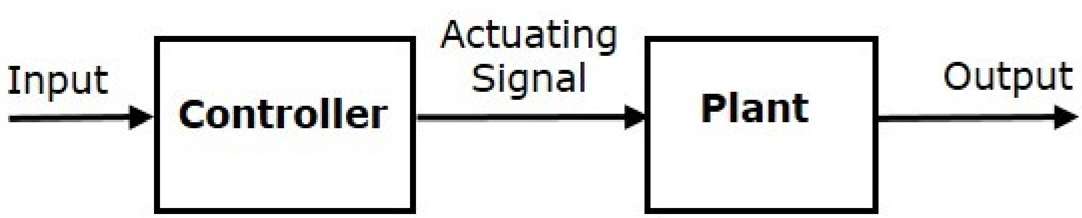
- **SISO** (Single Input and Single Output) control systems have one input and one output. Whereas, **MIMO** (Multiple Inputs and Multiple Outputs) control systems have more than one input and more than one output.

Open Loop and Closed Loop Control Systems

Control Systems can be classified as open loop control systems and closed loop control systems based on the **feedback path**.

In **open loop control systems**, output is not fed-back to the input. So, the control action is independent of the desired output.

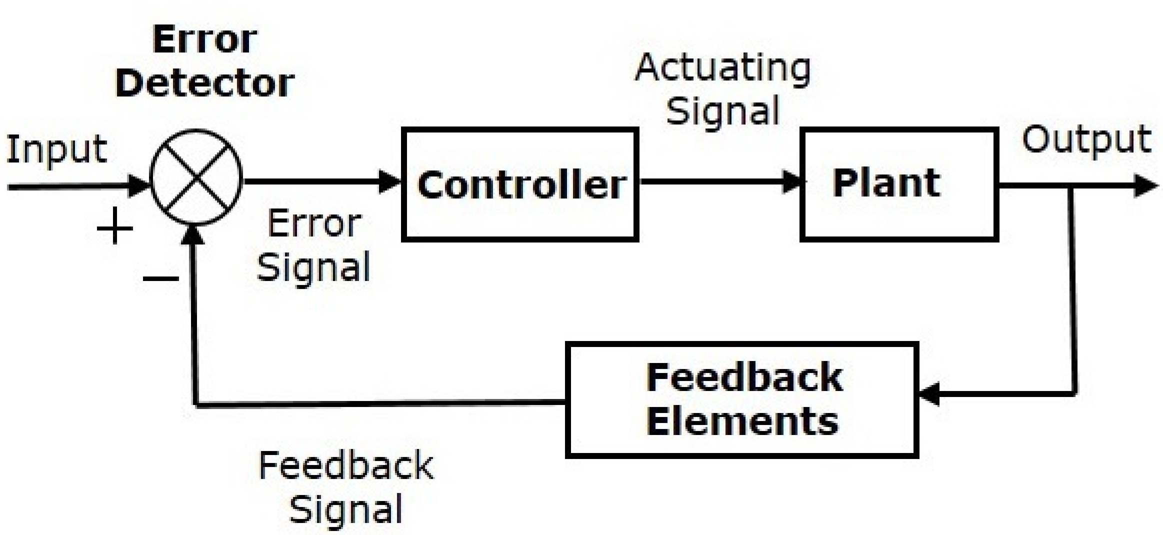
The following figure shows the block diagram of the open loop control system.



Here, an input is applied to a controller and it produces an actuating signal or controlling signal. This signal is given as an input to a plant or process which is to be controlled. So, the plant produces an output, which is controlled. The traffic lights control system which we discussed earlier is an example of an open loop control system.

In **closed loop control systems**, output is fed back to the input. So, the control action is dependent on the desired output.

The following figure shows the block diagram of negative feedback closed loop control system.



The error detector produces an error signal, which is the difference between the input and the feedback signal. This feedback signal is obtained from the block (feedback elements) by considering the output of the overall system as an input to this block. Instead of the direct input, the error signal is applied as an input to a controller.

So, the controller produces an actuating signal which controls the plant. In this combination, the output of the control system is adjusted automatically till we get the desired response. Hence, the closed loop control systems are also called the automatic control systems. Traffic lights control system having sensor at the input is an example of a closed loop control system.

The differences between the open loop and the closed loop control systems are mentioned in the following table.

Open Loop Control Systems	Closed Loop Control Systems
Control action is independent of the desired output.	Control action is dependent of the desired output.
Feedback path is not present.	Feedback path is present.
These are also called as **non-feedback control systems**.	These are also called as **feedback control systems**.
Easy to design.	Difficult to design.
These are economical.	These are costlier.
Inaccurate.	Accurate.

Control Systems - Feedback

If either the output or some part of the output is returned to the input side and utilized as part of the system input, then it is known as **feedback**. Feedback plays an important role in order to improve the performance of the control systems. In this chapter, let us discuss the types of feedback & effects of feedback.

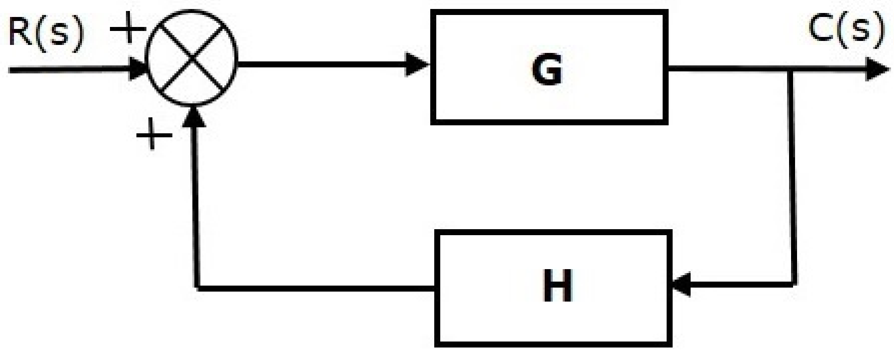
Types of Feedback

There are two types of feedback –

- Positive feedback
- Negative feedback

Positive Feedback

The positive feedback adds the reference input, $R(s)$ and feedback output. The following figure shows the block diagram of **positive feedback control system**.



The concept of transfer function will be discussed in later chapters. For the time being, consider the transfer function of positive feedback control system is,

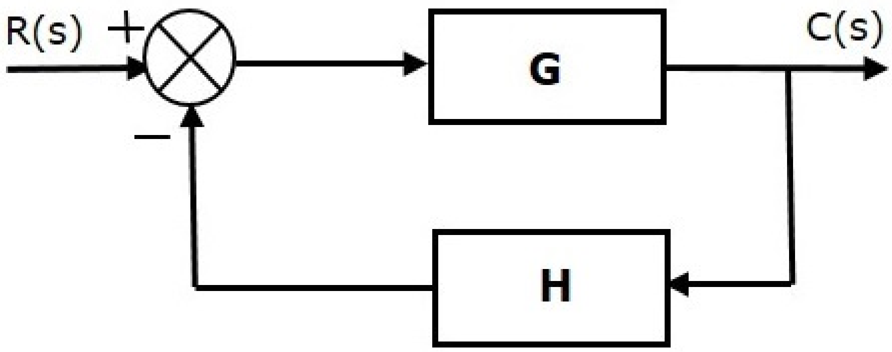
$$T = \frac{G}{1-GH} \qquad \text{(Equation 1)}$$

Where,

- **T** is the transfer function or overall gain of positive feedback control system.
- **G** is the open loop gain, which is function of frequency.
- **H** is the gain of feedback path, which is function of frequency.

Negative Feedback

Negative feedback reduces the error between the reference input, $R(s)$ and system output. The following figure shows the block diagram of the **negative feedback control system**.



Transfer function of negative feedback control system is,

$$T = \frac{G}{1+GH} \qquad \text{(Equation 2)}$$

Where,

- **T** is the transfer function or overall gain of negative feedback control system.
- **G** is the open loop gain, which is function of frequency.
- **H** is the gain of feedback path, which is function of frequency.

The derivation of the above transfer function is present in later chapters.

Effects of Feedback

Let us now understand the effects of feedback.

Effect of Feedback on Overall Gain

- From Equation 2, we can say that the overall gain of negative feedback closed loop control system is the ratio of 'G' and (1+GH). So, the overall gain may increase or decrease depending on the value of (1+GH).
- If the value of (1+GH) is less than 1, then the overall gain increases. In this case, 'GH' value is negative because the gain of the feedback path is negative.
- If the value of (1+GH) is greater than 1, then the overall gain decreases. In this case, 'GH' value is positive because the gain of the feedback path is positive.

In general, 'G' and 'H' are functions of frequency. So, the feedback will increase the overall gain of the system in one frequency range and decrease in the other frequency range.

Effect of Feedback on Sensitivity

Sensitivity of the overall gain of negative feedback closed loop control system (**T**) to the variation in open loop gain (**G**) is defined as

$$S_G^T = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{Percentage\ change\ in\ T}{Percentage\ change\ in\ G} \qquad \text{(Equation 3)}$$

Where, **∂T** is the incremental change in T due to incremental change in G.

We can rewrite Equation 3 as

$$S_G^T = \frac{\partial T}{\partial G}\frac{G}{T} \qquad \text{(Equation 4)}$$

Do partial differentiation with respect to G on both sides of Equation 2.

$$\frac{\partial T}{\partial G} = \frac{\partial}{\partial G}\left(\frac{G}{1+GH}\right) = \frac{(1+GH).1-G(H)}{(1+GH)^2} = \frac{1}{(1+GH)^2} \qquad \text{(Equation 5)}$$

From Equation 2, you will get

$$\frac{G}{T} = 1 + GH \qquad \text{(Equation 6)}$$

Substitute Equation 5 and Equation 6 in Equation 4.

$$S_G^T = \frac{1}{(1+GH)^2}(1+GH) = \frac{1}{1+GH}$$

So, we got the **sensitivity** of the overall gain of closed loop control system as the reciprocal of (1+GH). So, Sensitivity may increase or decrease depending on the value of (1+GH).

- If the value of (1+GH) is less than 1, then sensitivity increases. In this case, 'GH' value is negative because the gain of feedback path is negative.
- If the value of (1+GH) is greater than 1, then sensitivity decreases. In this case, 'GH' value is positive because the gain of feedback path is positive.

In general, 'G' and 'H' are functions of frequency. So, feedback will increase the sensitivity of the system gain in one frequency range and decrease in the other frequency range. Therefore, we have to choose the values of 'GH' in such a way that the system is insensitive or less sensitive to parameter variations.

Effect of Feedback on Stability

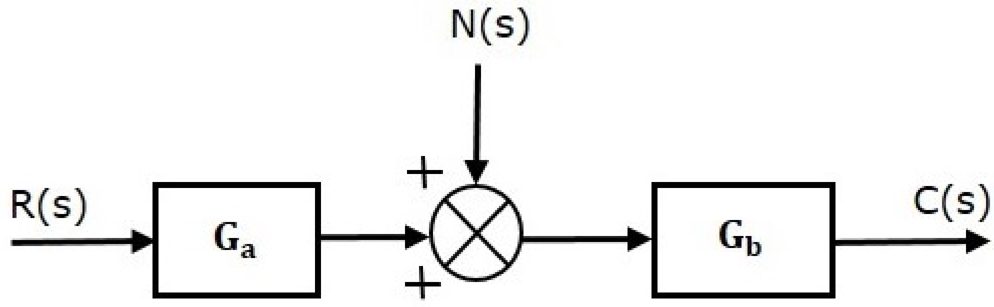
- A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable.
- In Equation 2, if the denominator value is zero (i.e., GH = -1), then the output of the control system will be infinite. So, the control system becomes unstable.

Therefore, we have to properly choose the feedback in order to make the control system stable.

Effect of Feedback on Noise

To know the effect of feedback on noise, let us compare the transfer function relations with and without feedback due to noise signal alone.

Consider an **open loop control system** with noise signal as shown below.

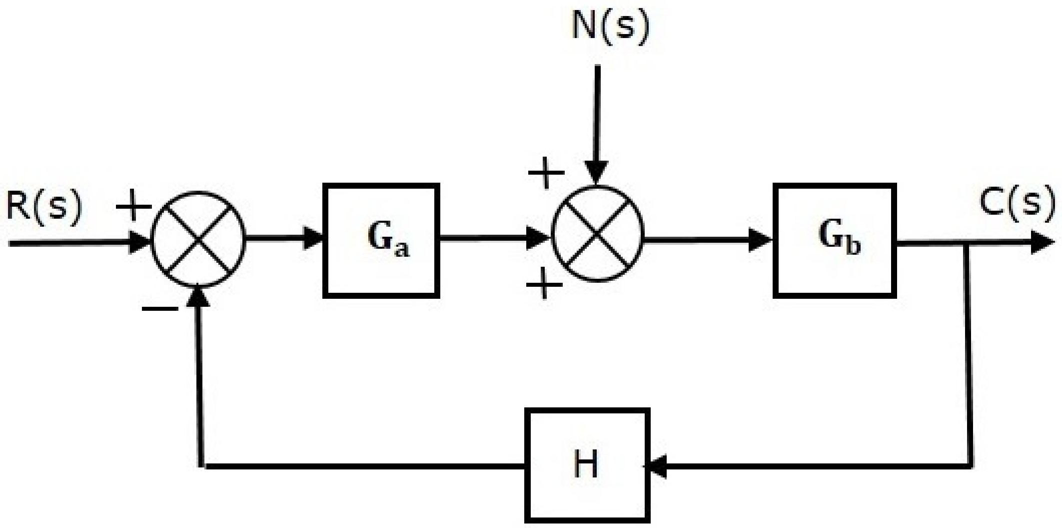


The **open loop transfer function** due to noise signal alone is

$$\frac{C(s)}{N(s)} = G_b \qquad \text{(Equation 7)}$$

It is obtained by making the other input $R(s)$ equal to zero.

Consider a **closed loop control system** with noise signal as shown below.



The **closed loop transfer function** due to noise signal alone is

$$\frac{C(s)}{N(s)} = \frac{G_b}{1 + G_a G_b H} \qquad \text{(Equation 8)}$$

It is obtained by making the other input $R(s)$ equal to zero.

Compare Equation 7 and Equation 8,

In the closed loop control system, the gain due to noise signal is decreased by a factor of $(1 + G_a G_b H)$ provided that the term $(1 + G_a G_b H)$ is greater than one.

Control Systems - Mathematical Models

The control systems can be represented with a set of mathematical equations known as **mathematical model**. These models are useful for analysis and design of control systems. Analysis of control system means finding the output when we know the input and mathematical model. Design of control system means finding the mathematical model when we know the input and the output.

The following mathematical models are mostly used.

- Differential equation model
- Transfer function model
- State space model

Let us discuss the first two models in this chapter.

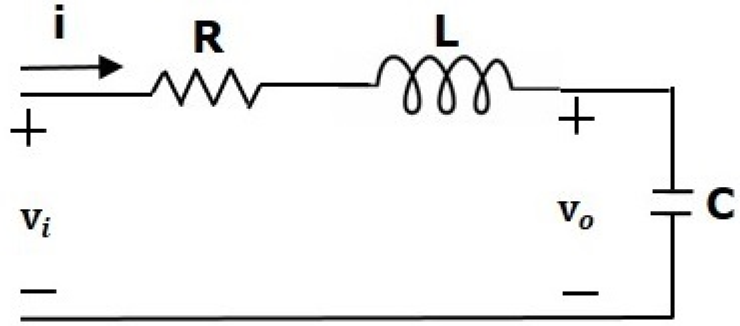
Differential Equation Model

Differential equation model is a time domain mathematical model of control systems. Follow these steps for differential equation model.

- Apply basic laws to the given control system.
- Get the differential equation in terms of input and output by eliminating the intermediate variable(s).

Example

Consider the following electrical system as shown in the following figure. This circuit consists of resistor, inductor and capacitor. All these electrical elements are connected in **series**. The input voltage applied to this circuit is v_i and the voltage across the capacitor is the output voltage v_o.



Mesh equation for this circuit is

$$v_i = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + v_o$$

Substitute, the current passing through capacitor $i = c\frac{\mathrm{d}v_o}{\mathrm{d}t}$ in the above equation.

$$\Rightarrow\ v_i = RC\frac{\mathrm{d}v_o}{\mathrm{d}t} + LC\frac{\mathrm{d}^2v_o}{\mathrm{d}t^2} + v_o$$

$$\Rightarrow \frac{\mathrm{d}^2v_o}{\mathrm{d}t^2} + \left(\frac{R}{L}\right)\frac{\mathrm{d}v_o}{\mathrm{d}t} + \left(\frac{1}{LC}\right)v_o = \left(\frac{1}{LC}\right)v_i$$

The above equation is a second order **differential equation**.

Transfer Function Model

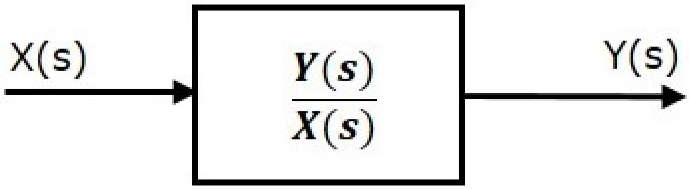
Transfer function model is an s-domain mathematical model of control systems. The **Transfer function** of a Linear Time Invariant (LTI) system is defined as the ratio of Laplace transform of output and Laplace transform of input by assuming all the initial conditions are zero.

If $x(t)$ and $y(t)$ are the input and output of an LTI system, then the corresponding Laplace transforms are $X(s)$ and $Y(s)$.

Therefore, the transfer function of LTI system is equal to the ratio of $Y(s)$ and $X(s)$.

$$i.e.,\ Transfer\ Function = \frac{Y(s)}{X(s)}$$

The transfer function model of an LTI system is shown in the following figure.



Here, we represented an LTI system with a block having transfer function inside it. And this block has an input $X(s)$ & output $Y(s)$.

Example

Previously, we got the differential equation of an electrical system as

$$\frac{\mathrm{d}^2 v_o}{\mathrm{d}t^2} + \left(\frac{R}{L}\right)\frac{\mathrm{d}v_o}{\mathrm{d}t} + \left(\frac{1}{LC}\right)v_o = \left(\frac{1}{LC}\right)v_i$$

Apply Laplace transform on both sides.

$$s^2 V_o(s) + \left(\frac{sR}{L}\right)V_o(s) + \left(\frac{1}{LC}\right)V_o(s) = \left(\frac{1}{LC}\right)V_i(s)$$

$$\Rightarrow \left\{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}\right\}V_o(s) = \left(\frac{1}{LC}\right)V_i(s)$$

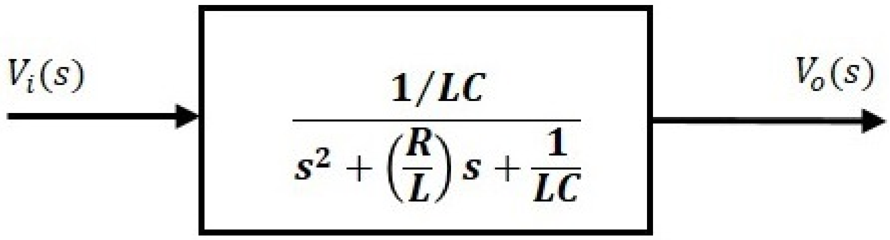
$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}$$

Where,

- $v_i(s)$ is the Laplace transform of the input voltage v_i

- $v_o(s)$ is the Laplace transform of the output voltage v_o

The above equation is a **transfer function** of the second order electrical system. The transfer function model of this system is shown below.



Here, we show a second order electrical system with a block having the transfer function inside it. And this block has an input $V_i(s)$ & an output $V_o(s)$.

Modelling of Mechanical Systems

In this chapter, let us discuss the **differential equation modeling** of mechanical systems. There are two types of mechanical systems based on the type of motion.

- Translational mechanical systems
- Rotational mechanical systems

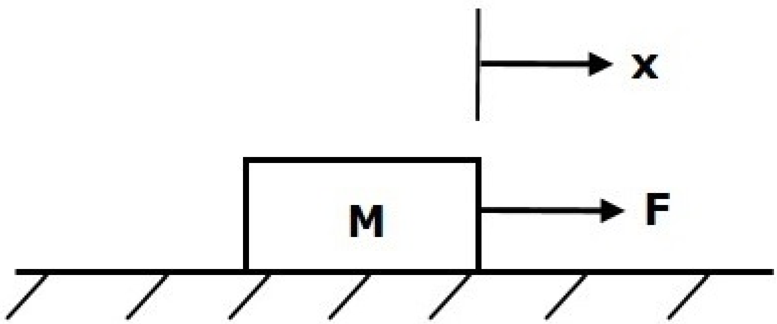
Modeling of Translational Mechanical Systems

Translational mechanical systems move along a **straight line**. These systems mainly consist of three basic elements. Those are mass, spring and dashpot or damper.

If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system. Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero. Let us now see the force opposed by these three elements individually.

Mass

Mass is the property of a body, which stores **kinetic energy**. If a force is applied on a body having mass **M**, then it is opposed by an opposing force due to mass. This opposing force is proportional to the acceleration of the body. Assume elasticity and friction are negligible.



$$F_m \propto a$$

$$\Rightarrow F_m = Ma = M\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$

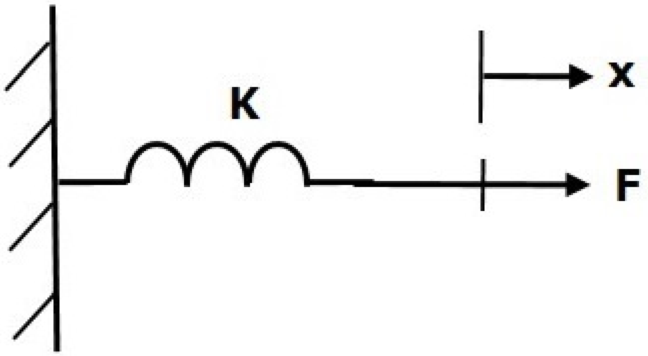
$$F = F_m = M\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$

Where,

- **F** is the applied force
- $\mathbf{F_m}$ is the opposing force due to mass
- **M** is mass
- **a** is acceleration
- **x** is displacement

Spring

Spring is an element, which stores **potential energy**. If a force is applied on spring **K**, then it is opposed by an opposing force due to elasticity of spring. This opposing force is proportional to the displacement of the spring. Assume mass and friction are negligible.



$$F \propto x$$

$$\Rightarrow F_k = Kx$$

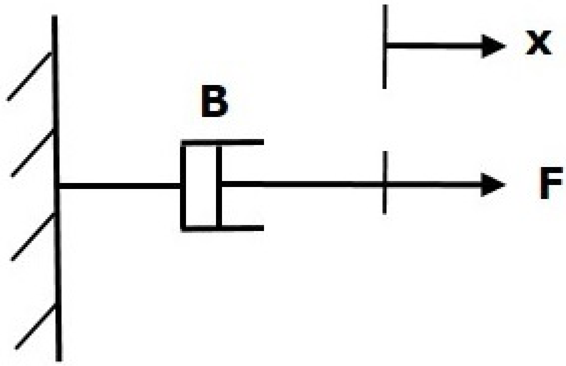
$$F = F_k = Kx$$

Where,

- **F** is the applied force
- $\mathbf{F_k}$ is the opposing force due to elasticity of spring
- **K** is spring constant
- **x** is displacement

Dashpot

If a force is applied on dashpot **B**, then it is opposed by an opposing force due to **friction** of the dashpot. This opposing force is proportional to the velocity of the body. Assume mass and elasticity are negligible.



$$F_b \propto \nu$$

$$\Rightarrow F_b = B\nu = B\frac{\mathrm{d}x}{\mathrm{d}t}$$

$$F = F_b = B\frac{\mathrm{d}x}{\mathrm{d}t}$$

Where,

- $\mathbf{F_b}$ is the opposing force due to friction of dashpot
- **B** is the frictional coefficient
- **v** is velocity
- **x** is displacement

Modeling of Rotational Mechanical Systems

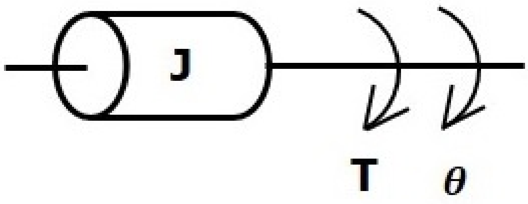
Rotational mechanical systems move about a fixed axis. These systems mainly consist of three basic elements. Those are **moment of inertia, torsional spring** and **dashpot**.

If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system. Since the applied torque and the opposing torques are in opposite directions, the algebraic sum of torques acting on the system is zero. Let us now see the torque opposed by these three elements individually.

Moment of Inertia

In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores **kinetic energy**.

If a torque is applied on a body having moment of inertia **J**, then it is opposed by an opposing torque due to the moment of inertia. This opposing torque is proportional to angular acceleration of the body. Assume elasticity and friction are negligible.



$$T_j \propto \alpha$$

$$\Rightarrow T_j = J\alpha = J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

$$T = T_j = J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

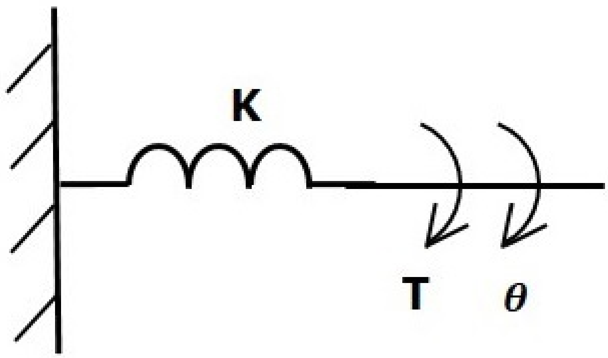
Where,

- **T** is the applied torque
- $\mathbf{T_j}$ is the opposing torque due to moment of inertia
- **J** is moment of inertia
- **α** is angular acceleration
- **θ** is angular displacement

Torsional Spring

In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores **potential energy**.

If a torque is applied on torsional spring **K**, then it is opposed by an opposing torque due to the elasticity of torsional spring. This opposing torque is proportional to the angular displacement of the torsional spring. Assume that the moment of inertia and friction are negligible.



$$T_k \propto \ \theta$$

$$\Rightarrow T_k = K\theta$$

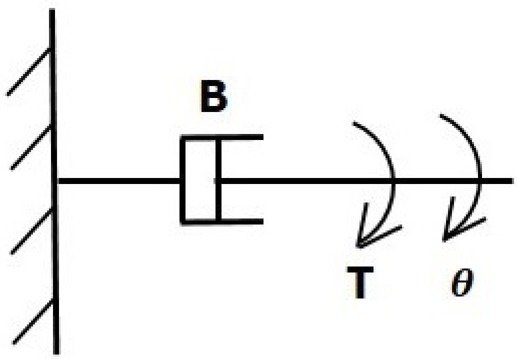
$$T = T_k = K\theta$$

Where,

- **T** is the applied torque
- $\mathbf{T_k}$ is the opposing torque due to elasticity of torsional spring
- **K** is the torsional spring constant
- **θ** is angular displacement

Dashpot

If a torque is applied on dashpot **B**, then it is opposed by an opposing torque due to the **rotational friction** of the dashpot. This opposing torque is proportional to the angular velocity of the body. Assume the moment of inertia and elasticity are negligible.



$$T_b \propto \omega$$

$$\Rightarrow T_b = B\omega = B\frac{\mathrm{d}\theta}{\mathrm{d}t}$$

$$T = T_b = B\frac{\mathrm{d}\theta}{\mathrm{d}t}$$

Where,

- **T_b** is the opposing torque due to the rotational friction of the dashpot
- **B** is the rotational friction coefficient
- **ω** is the angular velocity
- **θ** is the angular displacement

Electrical Analogies of Mechanical Systems

Two systems are said to be **analogous** to each other if the following two conditions are satisfied.

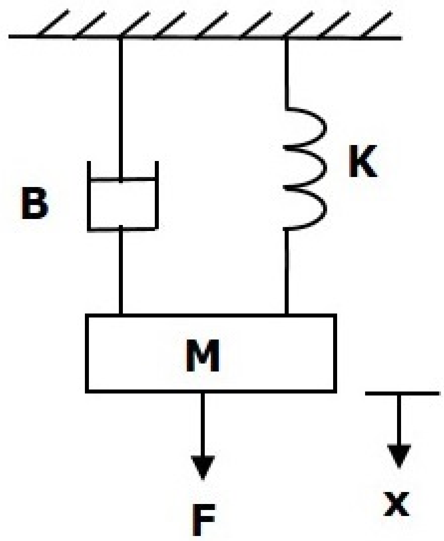
- The two systems are physically different
- Differential equation modelling of these two systems are same

Electrical systems and mechanical systems are two physically different systems. There are two types of electrical analogies of translational mechanical systems. Those are force voltage analogy and force current analogy.

Force Voltage Analogy

In force voltage analogy, the mathematical equations of **translational mechanical system** are compared with mesh equations of the electrical system.

Consider the following translational mechanical system as shown in the following figure.



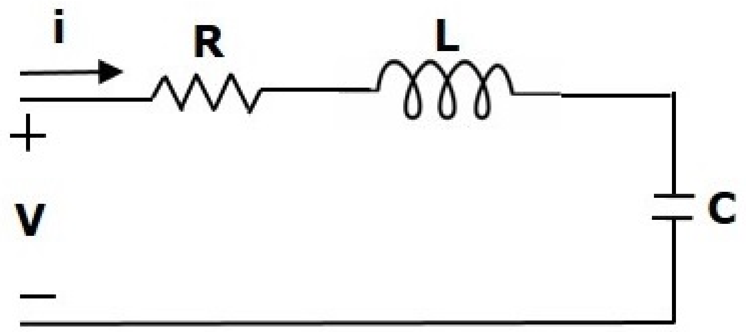
The **force balanced equation** for this system is

$$F = F_m + F_b + F_k$$

$$\Rightarrow F = M\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + B\frac{\mathrm{d}x}{\mathrm{d}t} + Kx \qquad \textbf{(Equation 1)}$$

Consider the following electrical system as shown in the following figure. This circuit consists of a resistor, an inductor and a capacitor. All these electrical elements are connected in a series.

The input voltage applied to this circuit is V volts and the current flowing through the circuit is i Amps.



Mesh equation for this circuit is

$$V = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{c}\int idt$$ **(Equation 2)**

Substitute, $i = \frac{\mathrm{d}q}{\mathrm{d}t}$ in Equation 2.

$$V = R\frac{\mathrm{d}q}{\mathrm{d}t} + L\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + \frac{q}{C}$$

$$\Rightarrow V = L\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + R\frac{\mathrm{d}q}{\mathrm{d}t} + \left(\frac{1}{c}\right)q$$ **(Equation 3)**

By comparing Equation 1 and Equation 3, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

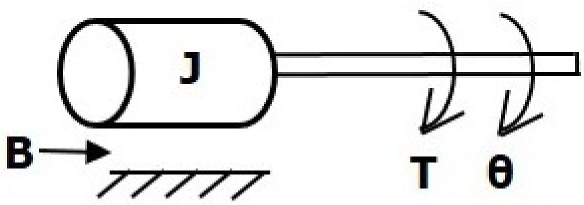
Translational Mechanical System	Electrical System
Force(F)	Voltage(V)
Mass(M)	Inductance(L)
Frictional Coefficient(B)	Resistance(R)
Spring Constant(K)	Reciprocal of Capacitance $(\frac{1}{c})$
Displacement(x)	Charge(q)
Velocity(v)	Current(i)

Similarly, there is torque voltage analogy for rotational mechanical systems. Let us now discuss about this analogy.

Torque Voltage Analogy

In this analogy, the mathematical equations of **rotational mechanical system** are compared with mesh equations of the electrical system.

Rotational mechanical system is shown in the following figure.



The torque balanced equation is

$$T = T_j + T_b + T_k$$

$$\Rightarrow T = J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + B\frac{\mathrm{d}\theta}{\mathrm{d}t} + k\theta$$ **(Equation 4)**

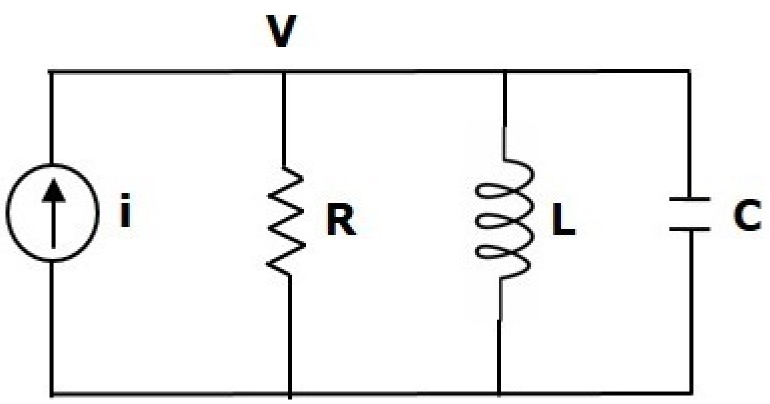
By comparing Equation 4 and Equation 3, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Voltage(V)
Moment of Inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance $(\frac{1}{c})$
Angular Displacement(θ)	Charge(q)
Angular Velocity(ω)	Current(i)

Force Current Analogy

In force current analogy, the mathematical equations of the **translational mechanical system** are compared with the nodal equations of the electrical system.

Consider the following electrical system as shown in the following figure. This circuit consists of current source, resistor, inductor and capacitor. All these electrical elements are connected in parallel.



The nodal equation is

$$i = \frac{V}{R} + \frac{1}{L}\int V dt + C\frac{\mathrm{d}V}{\mathrm{d}t} \qquad \textbf{(Equation 5)}$$

Substitute, $V = \frac{\mathrm{d}\Psi}{\mathrm{d}t}$ in Equation 5.

$$i = \frac{1}{R}\frac{\mathrm{d}\Psi}{\mathrm{d}t} + \left(\frac{1}{L}\right)\Psi + C\frac{\mathrm{d}^2\Psi}{\mathrm{d}t^2}$$

$$\Rightarrow i = C\frac{\mathrm{d}^2\Psi}{\mathrm{d}t^2} + \left(\frac{1}{R}\right)\frac{\mathrm{d}\Psi}{\mathrm{d}t} + \left(\frac{1}{L}\right)\Psi \qquad \textbf{(Equation 6)}$$

By comparing Equation 1 and Equation 6, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Current(i)
Mass(M)	Capacitance(C)
Frictional coefficient(B)	Reciprocal of Resistance $\left(\frac{1}{R}\right)$
Spring constant(K)	Reciprocal of Inductance $\left(\frac{1}{L}\right)$
Displacement(x)	Magnetic Flux(ψ)
Velocity(v)	Voltage(V)

Similarly, there is a torque current analogy for rotational mechanical systems. Let us now discuss this analogy.

Torque Current Analogy

In this analogy, the mathematical equations of the **rotational mechanical system** are compared with the nodal mesh equations of the electrical system.

By comparing Equation 4 and Equation 6, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Current(i)
Moment of inertia(J)	Capacitance(C)
Rotational friction coefficient(B)	Reciprocal of Resistance $\left(\frac{1}{R}\right)$
Torsional spring constant(K)	Reciprocal of Inductance $\left(\frac{1}{L}\right)$
Angular displacement(θ)	Magnetic flux(ψ)
Angular velocity(ω)	Voltage(V)

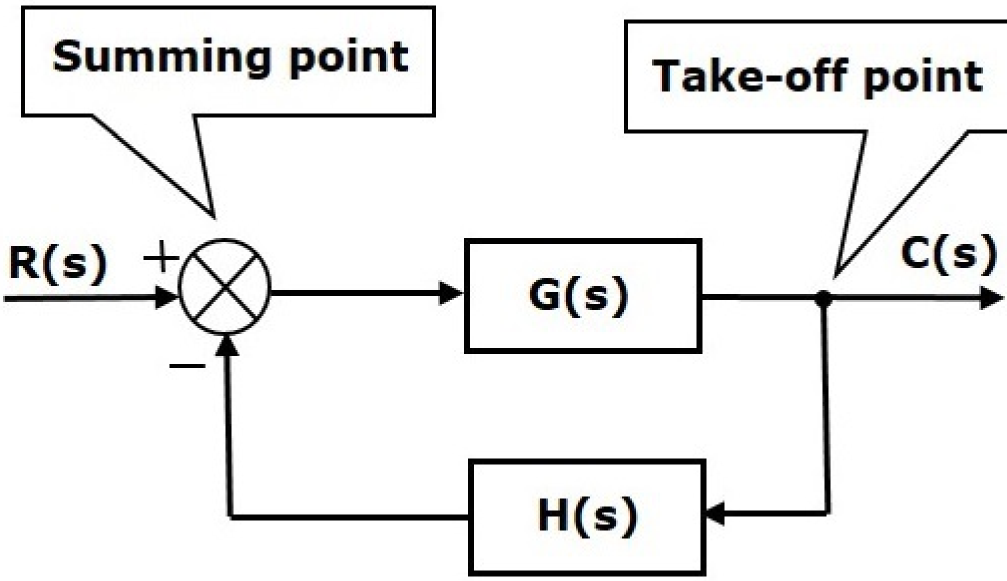
In this chapter, we discussed the electrical analogies of the mechanical systems. These analogies are helpful to study and analyze the non-electrical system like mechanical system from analogous electrical system.

Control Systems - Block Diagrams

Block diagrams consist of a single block or a combination of blocks. These are used to represent the control systems in pictorial form.

Basic Elements of Block Diagram

The basic elements of a block diagram are a block, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following figure to identify these elements.



The above block diagram consists of two blocks having transfer functions G(s) and H(s). It is also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals. Let us now discuss these elements one by one.

Block

The transfer function of a component is represented by a block. Block has single input and single output.

The following figure shows a block having input X(s), output Y(s) and the transfer function G(s).

Transfer Function, $G(s) = \frac{Y(s)}{X(s)}$

$$\Rightarrow Y(s) = G(s)X(s)$$

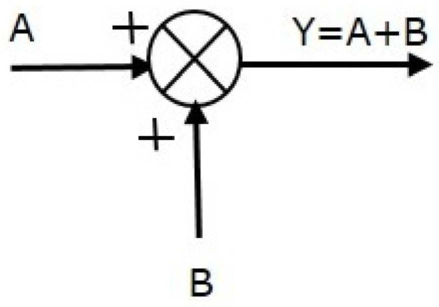
Output of the block is obtained by multiplying transfer function of the block with input.

Summing Point

The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs. Let us see these three operations one by one.

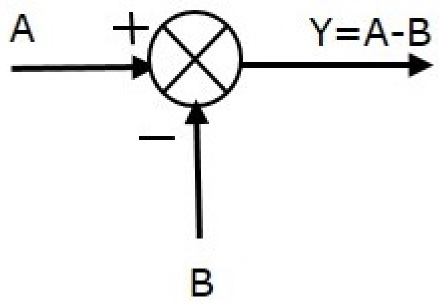
The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y as **sum of A and B.**

i.e.,Y = A + B.



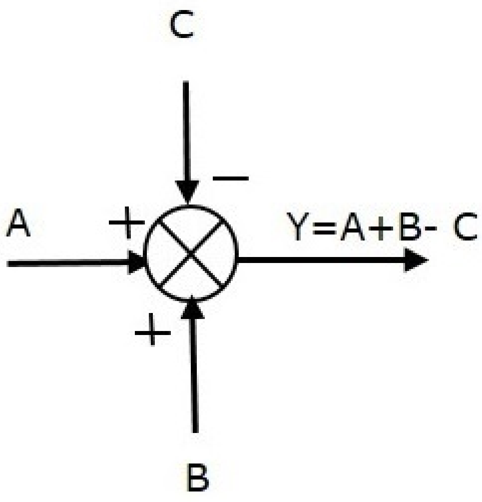
The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B are having opposite signs, i.e., A is having positive sign and B is having negative sign. So, the summing point produces the output **Y** as the **difference of A and B**.

Y = A + (-B) = A - B.



The following figure shows the summing point with three inputs (A, B, C) and one output (Y). Here, the inputs A and B are having positive signs and C is having a negative sign. So, the summing point produces the output **Y** as

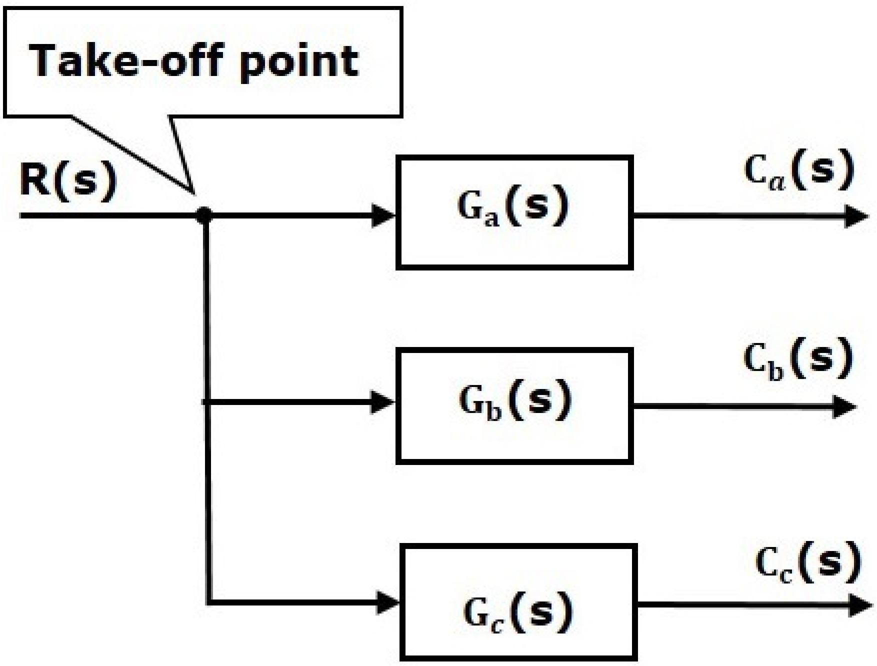
Y = A + B + (−C) = A + B − C.



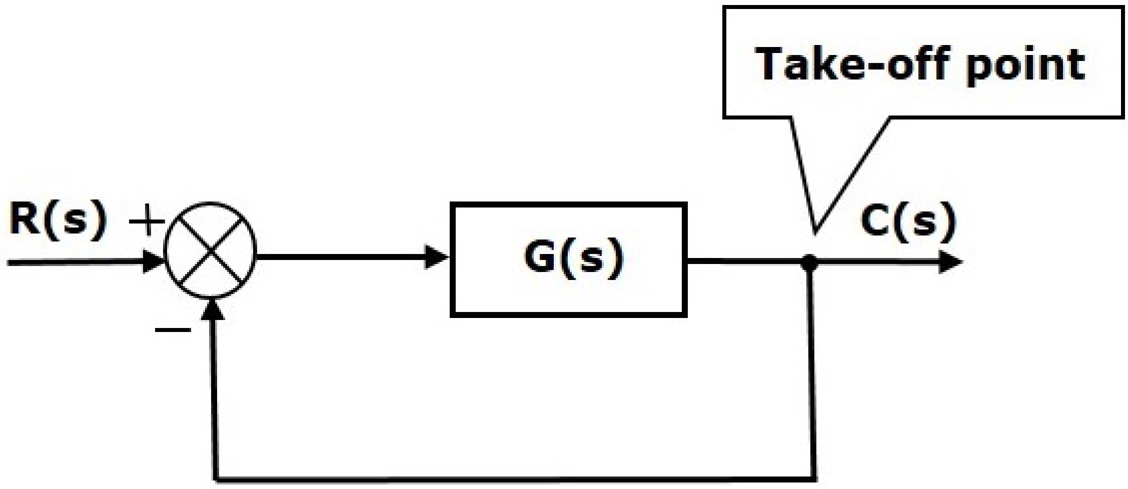
Take-off Point

The take-off point is a point from which the same input signal can be passed through more than one branch. That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.

In the following figure, the take-off point is used to connect the same input, R(s) to two more blocks.



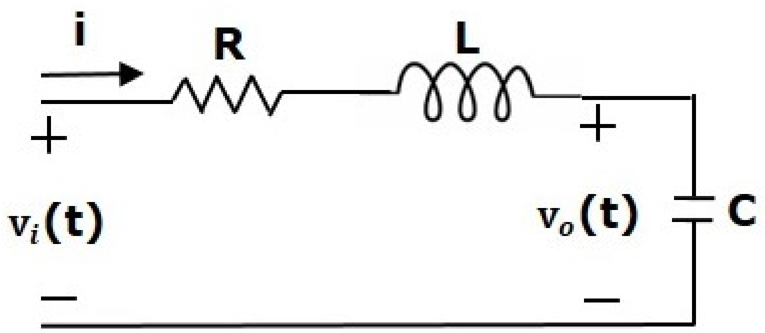
In the following figure, the take-off point is used to connect the output C(s), as one of the inputs to the summing point.



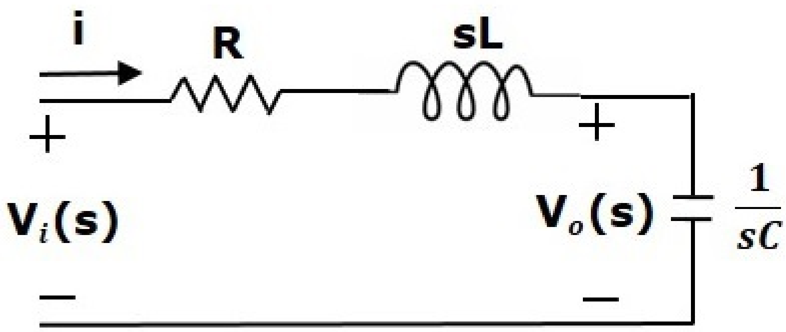
Block Diagram Representation of Electrical Systems

In this section, let us represent an electrical system with a block diagram. Electrical systems contain mainly three basic elements — **resistor, inductor and capacitor**.

Consider a series of RLC circuit as shown in the following figure. Where, $V_i(t)$ and $V_o(t)$ are the input and output voltages. Let i(t) be the current passing through the circuit. This circuit is in time domain.



By applying the Laplace transform to this circuit, will get the circuit in s-domain. The circuit is as shown in the following figure.



From the above circuit, we can write

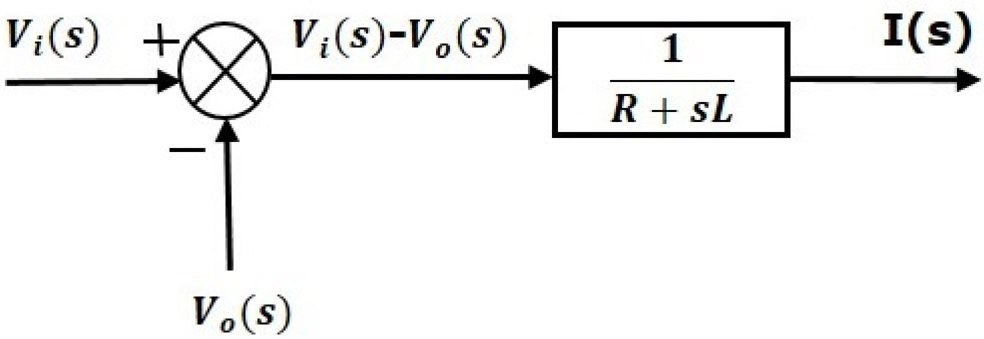
$$I(s) = \frac{V_i(s) - V_o(s)}{R + sL}$$

$$\Rightarrow I(s) = \left\{\frac{1}{R+sL}\right\}\{V_i(s) - V_o(s)\} \qquad \textbf{(Equation 1)}$$

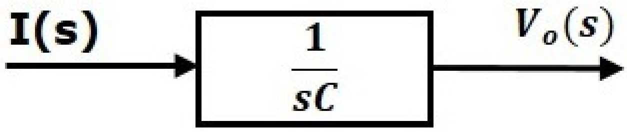
$$V_o(s) = \left(\frac{1}{sC}\right) I(s) \qquad \textbf{(Equation 2)}$$

Let us now draw the block diagrams for these two equations individually. And then combine those block diagrams properly in order to get the overall block diagram of series of RLC Circuit (s-domain).

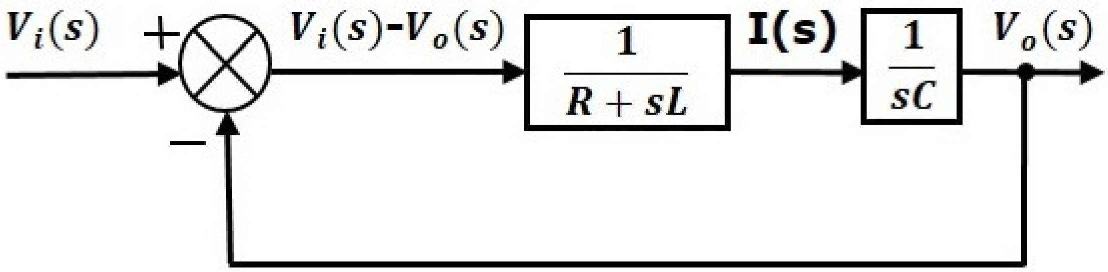
Equation 1 can be implemented with a block having the transfer function, $\frac{1}{R+sL}$. The input and output of this block are $\{V_i(s) - V_o(s)\}$ and $I(s)$. We require a summing point to get $\{V_i(s) - V_o(s)\}$. The block diagram of Equation 1 is shown in the following figure.



Equation 2 can be implemented with a block having transfer function, $\frac{1}{sC}$. The input and output of this block are $I(s)$ and $V_o(s)$. The block diagram of Equation 2 is shown in the following figure.



The overall block diagram of the series of RLC Circuit (s-domain) is shown in the following figure.



Similarly, you can draw the **block diagram** of any electrical circuit or system just by following this simple procedure.

- Convert the time domain electrical circuit into an s-domain electrical circuit by applying Laplace transform.
- Write down the equations for the current passing through all series branch elements and voltage across all shunt branches.
- Draw the block diagrams for all the above equations individually.
- Combine all these block diagrams properly in order to get the overall block diagram of the electrical circuit (s-domain).

Control Systems - Block Diagram Algebra

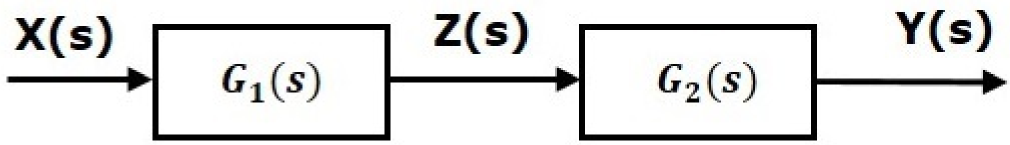
Block diagram algebra is nothing but the algebra involved with the basic elements of the block diagram. This algebra deals with the pictorial representation of algebraic equations.

Basic Connections for Blocks

There are three basic types of connections between two blocks.

Series Connection

Series connection is also called **cascade connection**. In the following figure, two blocks having transfer functions $G_1(s)$ and $G_2(s)$ are connected in series.



For this combination, we will get the output $Y(s)$ as

$$Y(s) = G_2(s)Z(s)$$

Where, $Z(s) = G_1(s)X(s)$

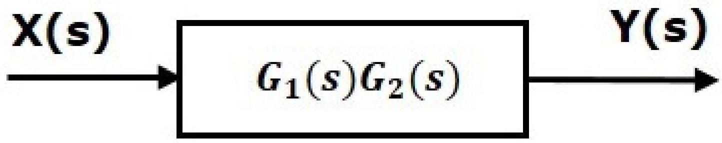
$$\Rightarrow Y(s) = G_2(s)[G_1(s)X(s)]$$
$$= G_1(s)G_2(s)X(s)$$

$$\Rightarrow Y(s) = \{G_1(s)G_2(s)\}X(s)$$

Compare this equation with the standard form of the output equation, $Y(s) = G(s)X(s)$.

Where, $G(s) = G_1(s)G_2(s)$.

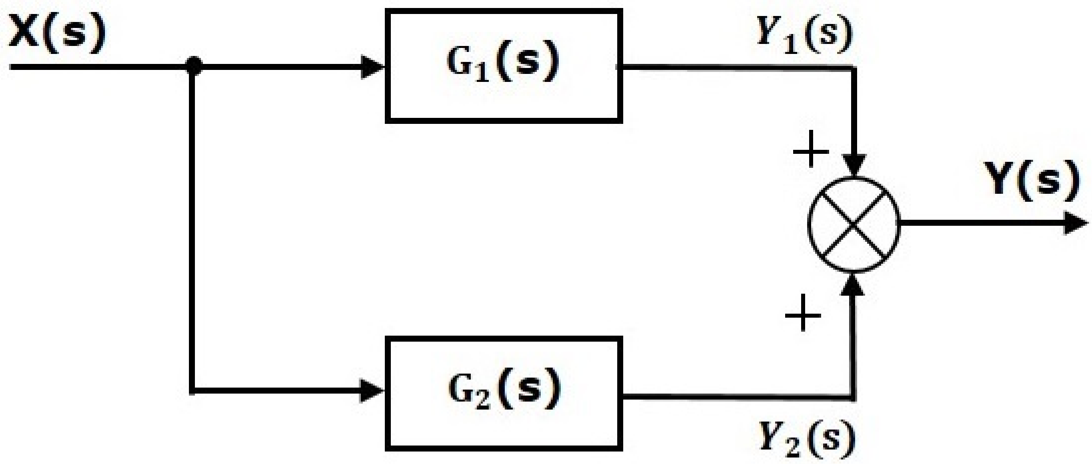
That means we can represent the **series connection** of two blocks with a single block. The transfer function of this single block is the **product of the transfer functions** of those two blocks. The equivalent block diagram is shown below.



Similarly, you can represent series connection of 'n' blocks with a single block. The transfer function of this single block is the product of the transfer functions of all those 'n' blocks.

Parallel Connection

The blocks which are connected in **parallel** will have the **same input**. In the following figure, two blocks having transfer functions $G_1(s)$ and $G_2(s)$ are connected in parallel. The outputs of these two blocks are connected to the summing point.



For this combination, we will get the output $Y(s)$ as

$$Y(s) = Y_1(s) + Y_2(s)$$

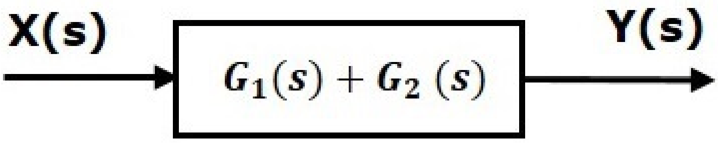
Where, $Y_1(s) = G_1(s)X(s)$ and $Y_2(s) = G_2(s)X(s)$

$$\Rightarrow Y(s) = G_1(s)X(s) + G_2(s)X(s)$$
$$= \{G_1(s) + G_2(s)\}X(s)$$

Compare this equation with the standard form of the output equation, $Y(s) = G(s)X(s)$.

Where, $G(s) = G_1(s) + G_2(s)$.

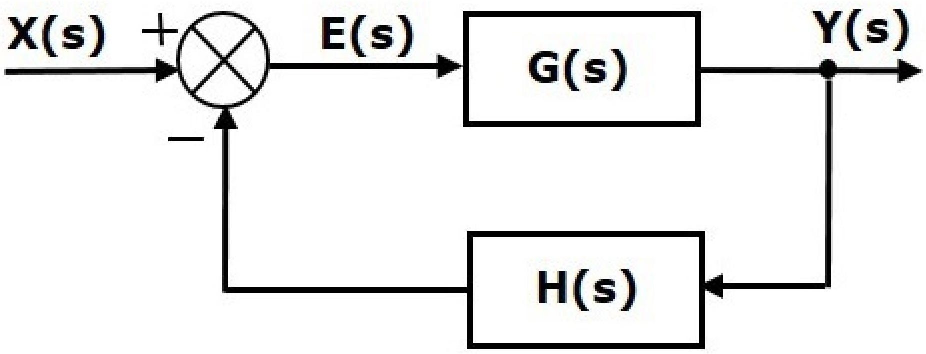
That means we can represent the **parallel connection** of two blocks with a single block. The transfer function of this single block is the **sum of the transfer functions** of those two blocks. The equivalent block diagram is shown below.



Similarly, you can represent parallel connection of 'n' blocks with a single block. The transfer function of this single block is the algebraic sum of the transfer functions of all those 'n' blocks.

Feedback Connection

As we discussed in previous chapters, there are two types of **feedback** — positive feedback and negative feedback. The following figure shows negative feedback control system. Here, two blocks having transfer functions $G(s)$ and $H(s)$ form a closed loop.



The output of the summing point is -

$$E(s) = X(s) - H(s)Y(s)$$

The output $Y(s)$ is -

$$Y(s) = E(s)G(s)$$

Substitute $E(s)$ value in the above equation.

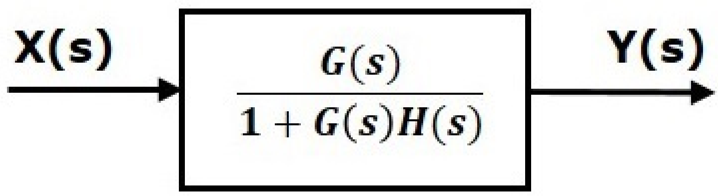
$$Y(s) = \{X(s) - H(s)Y(s)\}G(s)\}$$

$$Y(s)\{1 + G(s)H(s)\} = X(s)G(s)\}$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Therefore, the negative feedback closed loop transfer function is $\frac{G(s)}{1+G(s)H(s)}$

This means we can represent the negative feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the negative feedback. The equivalent block diagram is shown below.



Similarly, you can represent the positive feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the positive feedback, i.e., $\frac{G(s)}{1-G(s)H(s)}$

Block Diagram Algebra for Summing Points

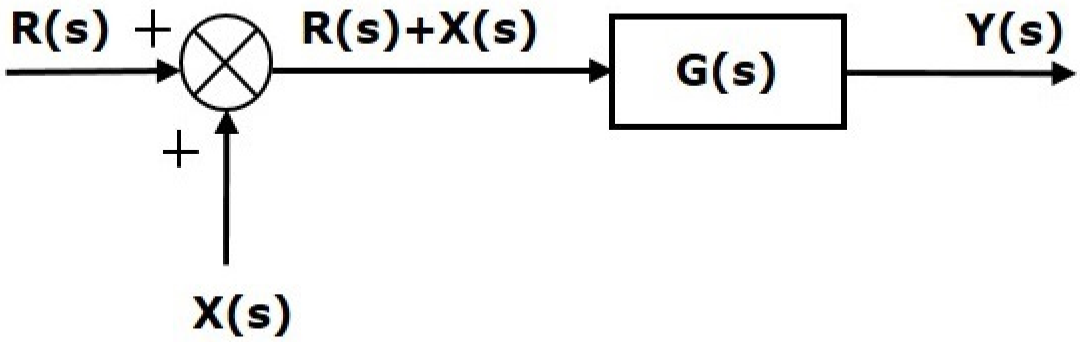
There are two possibilities of shifting summing points with respect to blocks –

- Shifting summing point after the block
- Shifting summing point before the block

Let us now see what kind of arrangements need to be done in the above two cases one by one.

Shifting Summing Point After the Block

Consider the block diagram shown in the following figure. Here, the summing point is present before the block.



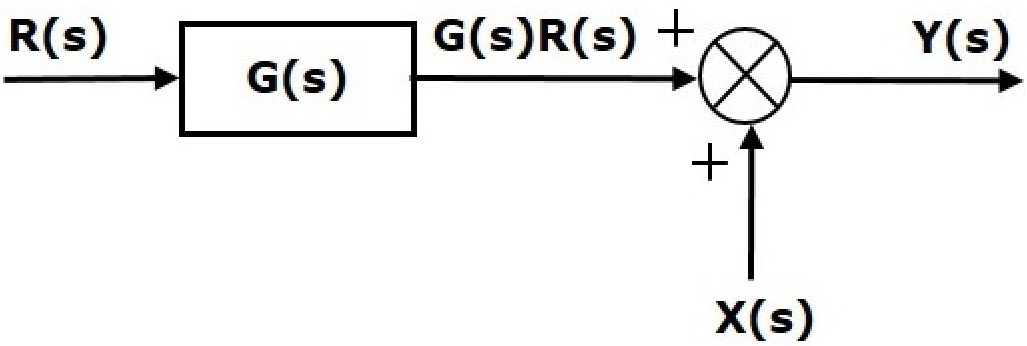
Summing point has two inputs $R(s)$ and $X(s)$. The output of it is $\{R(s)+X(s)\}$.

So, the input to the block $G(s)$ is $\{R(s)+X(s)\}$ and the output of it is –

$$Y(s)=G(s)\{R(s)+X(s)\}$$

$$\Rightarrow Y(s)=G(s)R(s)+G(s)X(s)$$ **(Equation 1)**

Now, shift the summing point after the block. This block diagram is shown in the following figure.



Output of the block $G(s)$ is $G(s)R(s)$.

The output of the summing point is

$$Y(s)=G(s)R(s)+X(s)$$ **(Equation 2)**

Compare Equation 1 and Equation 2.

Control Systems - Block Diagram Reduction

The concepts discussed in the previous chapter are helpful for reducing (simplifying) the block diagrams.

Block Diagram Reduction Rules

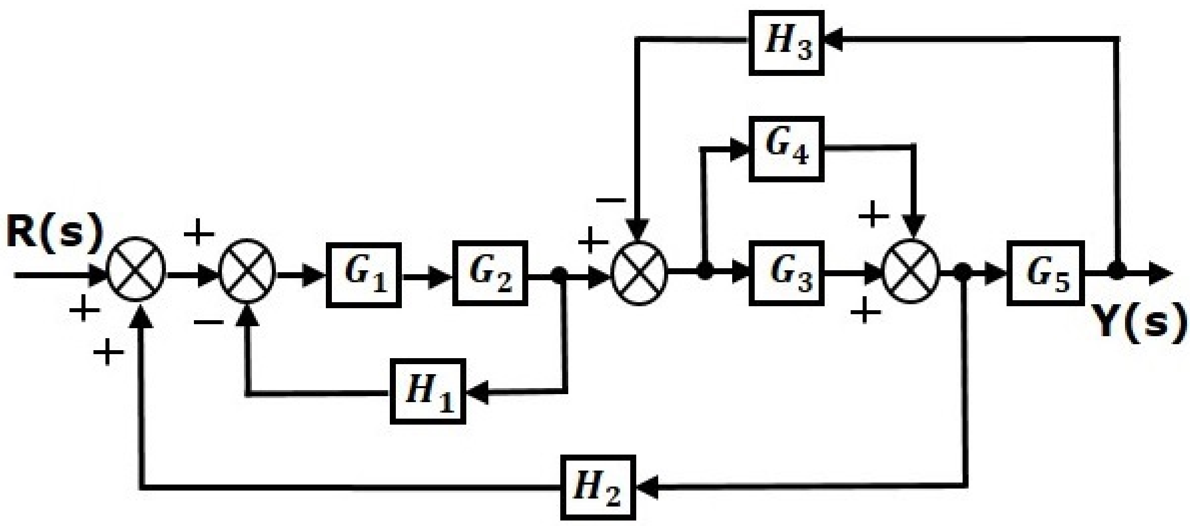
Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.

- **Rule 1** – Check for the blocks connected in series and simplify.
- **Rule 2** – Check for the blocks connected in parallel and simplify.
- **Rule 3** – Check for the blocks connected in feedback loop and simplify.
- **Rule 4** – If there is difficulty with take-off point while simplifying, shift it towards right.
- **Rule 5** – If there is difficulty with summing point while simplifying, shift it towards left.
- **Rule 6** – Repeat the above steps till you get the simplified form, i.e., single block.

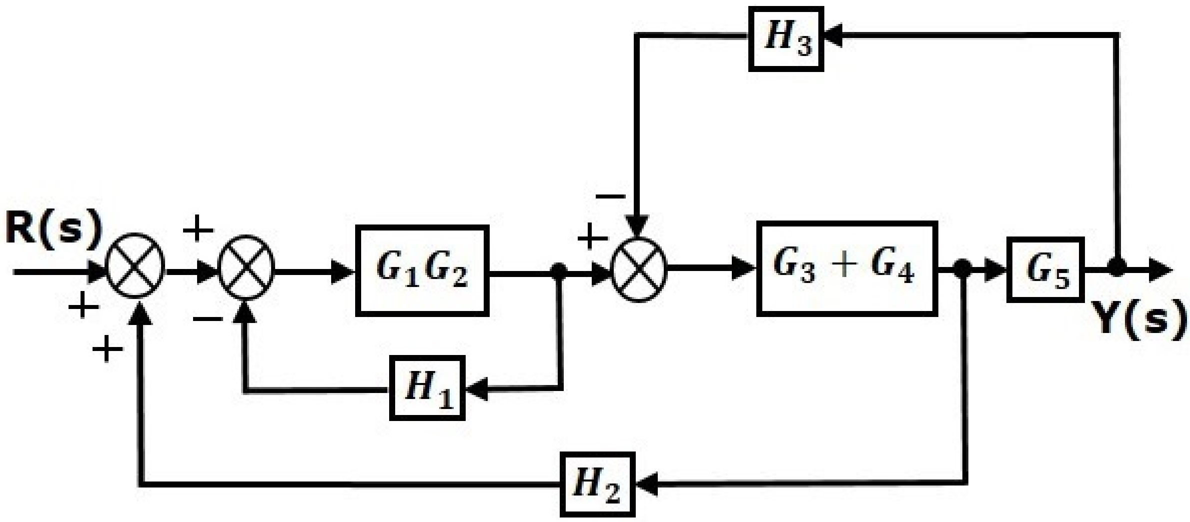
Note – The transfer function present in this single block is the transfer function of the overall block diagram.

Example

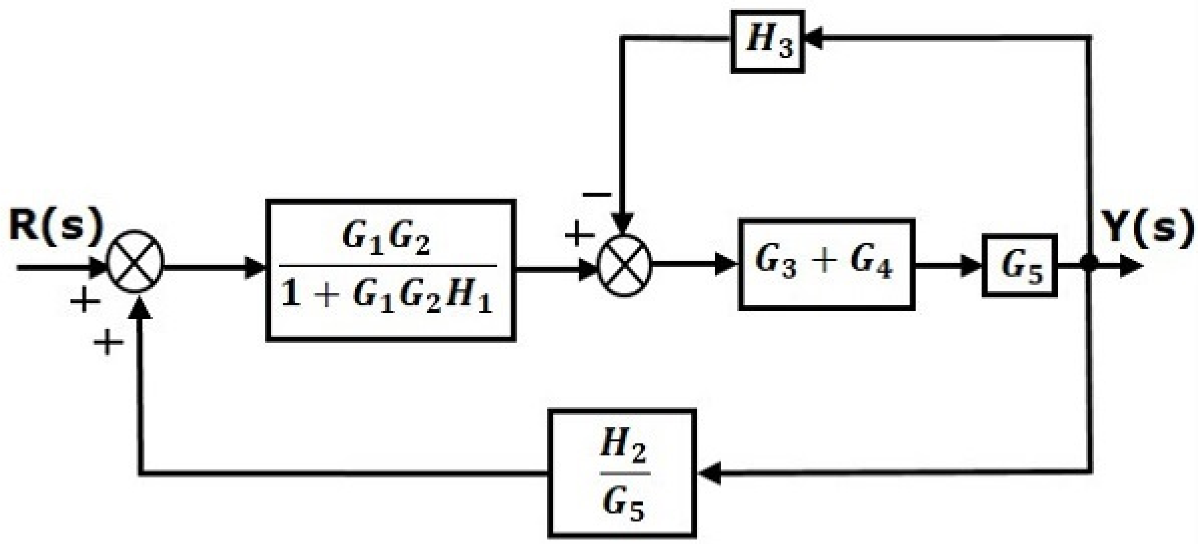
Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



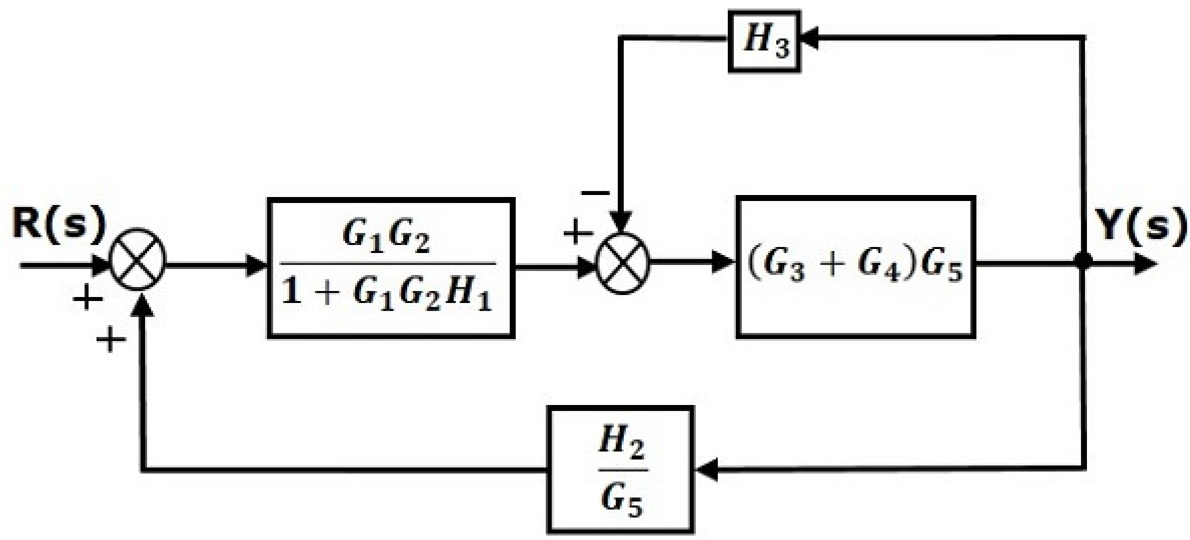
Step 1 – Use Rule 1 for blocks G_1 and G_2. Use Rule 2 for blocks G_3 and G_4. The modified block diagram is shown in the following figure.



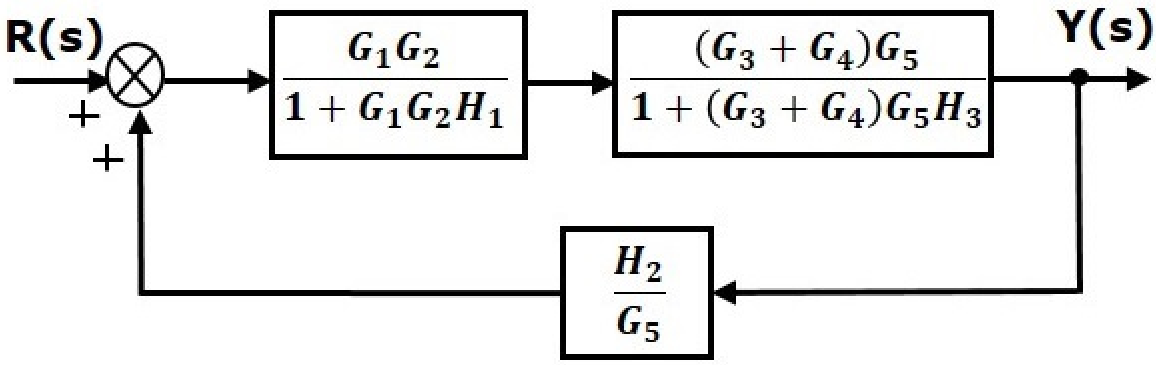
Step 2 – Use Rule 3 for blocks G_1G_2 and H_1. Use Rule 4 for shifting take-off point after the block G_5. The modified block diagram is shown in the following figure.



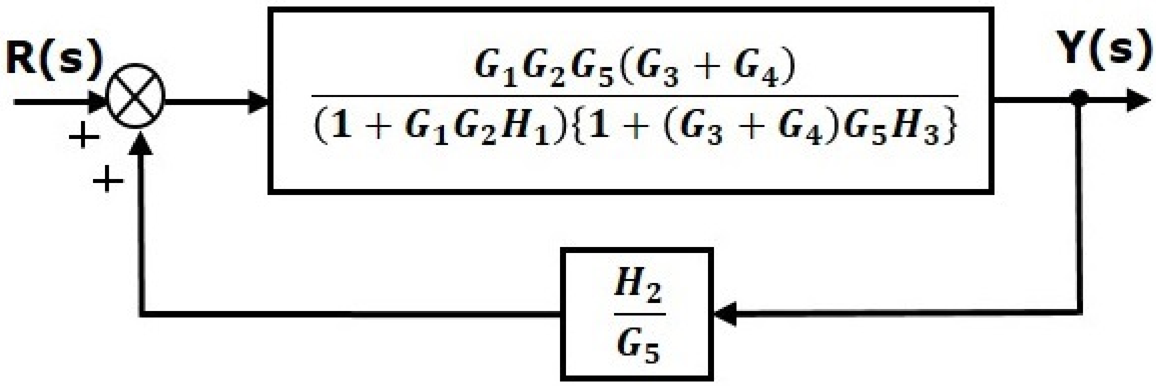
Step 3 – Use Rule 1 for blocks (G_3 + G_4) and G_5. The modified block diagram is shown in the following figure.



Step 4 – Use Rule 3 for blocks $(G_3 + G_4)G_5$ and H_3. The modified block diagram is shown in the following figure.



Step 5 – Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.



Step 6 – Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.

Therefore, the transfer function of the system is

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_5^2(G_3 + G_4)}{(1 + G_1G_2H_1)\{1 + (G_3 + G_4)G_5H_3\}G_5 - G_1G_2G_5(G_3 + G_4)H_2}$$

Note – Follow these steps in order to calculate the transfer function of the block diagram having multiple inputs.

- **Step 1** – Find the transfer function of block diagram by considering one input at a time and make the remaining inputs as zero.
- **Step 2** – Repeat step 1 for remaining inputs.
- **Step 3** – Get the overall transfer function by adding all those transfer functions.

The block diagram reduction process takes more time for complicated systems. Because, we have to draw the (partially simplified) block diagram after each step. So, to overcome this drawback, use signal flow graphs (representation).

In the next two chapters, we will discuss about the concepts related to signal flow graphs, i.e., how to represent signal flow graph from a given block diagram and calculation of transfer function just by using a gain formula without doing any reduction process.