

**LAGOS CITY POLYTECHNIC, IKEJA**  
**SCHOOL OF ENGINEERING AND APPLIED SCIENCE**  
**DEPARTMENT OF COMPUTER SCIENCE**  
**2016/2017 SEMESTER EXAMINATION**

<b>COURSE TITLE:</b> STATISTICAL THEORY II	<b>NO OF QUESTIONS :</b> 6
<b>COURSE CODE:</b> STA 321	<b>TIME ALLOWED:</b> 2 HRS
<b>FOR WHOM:</b> HND YR I CS	<b>PT</b>
<b>ANSWER ANY</b>	<b>INSTRUCTIONS:</b>

**FOUR QUESTIONS**

1. Let  $X$  follow a Binomial distribution with parameters  $n$  and  $p$ 
  - (a) Obtain the maximum likelihood estimator of  $p$
  - (b) Given the observations 2, 3, 6, 8 and 11, obtain the maximum likelihood estimate of  $p$ .
  
2. (a) Define the followings:
  - (i) Efficient estimator
  - (ii) Unbiased estimator  
 (b) Let  $X_1, X_2, X_3, \dots, X_n$ , be simple random sample from  $N(\mu, \sigma^2)$ , show that
  - (i)  $\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$  is an unbiased consistent estimator
  - (ii)  $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$
  
3. (a) Given the Fisher Information  $I(\theta)$ , show that:
  - (i)  $E[\lambda(x, \theta)] = 0$
  - (ii)  $I(\theta) = E[\lambda(x, \theta)]^2$
  - (iii)  $I(\theta) = -E[\lambda(x, \theta)]$  
 (b) Let  $X$  follows a Poisson distribution with parameter  $\lambda$ , obtain the Fisher Information of  $\lambda$ .
  
4. (a) State the factorization theorem
  - (b) Given a simple random sample Let  $X_1, X_2, X_3, \dots, X_n$  from a Bernoulli distribution with parameters  $1$  and  $P$ , show that  $T = \sum_{i=1}^n X_i$  is sufficient for  $P$ .
  
5. Given a continuous variable  $X$  to follow a Gamma distribution with parameters  $\alpha$  and  $\beta$ 
  - (a) Write down explicitly the probability density function of  $X$
  - (b) Obtain  $E(X)$ ,  $E(X^2)$  and  $V(X)$
  - (c) State the conditions under which the Gamma distribution turns to Chi-square distribution and hence determine  $E(X)$ ,  $E(X^2)$  and  $V(X)$
  
6. Given a continuous variable  $X$  to follow a Beta distribution with parameters  $\alpha$  and  $\beta$ 
  - (a) Write down explicitly the probability density function of  $X$
  - (b) Determine the mean of  $X$  and variance of  $X$