

LAGOS CITY POLYTECHNIC, IKEJA

SCHOOL OF MANAGEMENT AND BUSINESS STUDIES

DEPARTMENT OF GENERAL STUDIES

2015/2016 SECOND SEMESTER EXAMINATION

1. Given the following joint pdf of two random variables X and Y, establish the necessary and sufficient conditions for the independence of the random variables X and Y.

xy^2

COURSE TITLE: STATISTICAL THEORY II **NO. OF QUESTIONS:** 6
COURSE CODE: STA 321 **TIME ALLOWED:** 18
2 1/2 HRS

FOR WHOM: HND I CS ≥ E/T

INSTRUCTIONS:

Answer f(x, y) = 0 otherwise

1. Given the following joint pdf of two random variables X and Y, establish the necessary and sufficient conditions for the independence of the random variables X and Y.

(a) Given that X_1 and X_2 are two independent random variables, show that $X_1 + X_2$ is a random variable with probabilities $f(x, y) = 0$ otherwise

(b) Given that $X = (X_1, X_2, X_3, \dots, X_n)$ are is a random variable with probabilities $f(x, y) = 1/6 e^{-(2x+3y)}$; $x \geq 0, y \geq 0$ otherwise

2. Find the characteristic function of $X = X_1 + X_2 + \dots + X_n$ where X_1, X_2, \dots, X_n are two independent random variables, show that $\phi_{X_1 + X_2} = \phi_{X_1} \phi_{X_2}$

(b) Given that $X = (X_1, X_2, X_3, \dots, X_n)$ are is a random variable with probabilities $f(x, y) = 1/6 e^{-(2x+3y)}$; $x \geq 0, y \geq 0$ otherwise

3. (a) Define the following:
 (i) Likelihood function (ii) Maximum likelihood estimator

(b) Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from Bernoulli distribution with parameter p, that is, $f(x, p) = p^x (1-p)^{1-x}$, $x = 0, 1$ and zero elsewhere. Find the characteristic function of $X = X_1 + X_2 + \dots + X_n$

(i) Obtain the MLE, p of p if $x_1 = 4, x_2 = 7, x_3 = 5, x_4 = 3.5$ and $x_5 = 6$, obtain p

4. Given that X is a continuous random variable Gamma distribution with parameters α and β .

(a) Define the pdf of following:
 (i) Likelihood function (ii) Maximum likelihood estimator

(b) For what values of a and b does the Gamma distribution become Chi-square distribution? Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from Exponential distribution with parameter p, that is, $f(x, p) = p e^{-px}$, $x = 0, 1$ and zero elsewhere.

(c) Derive the mean of Gamma distribution and hence obtain the mean of Chi-square distribution
 (i) Obtain the MLE, p of p (ii) if $x_1 = 4, x_2 = 7, x_3 = 5, x_4 = 3.5$ and $x_5 = 6$, obtain p

5. (a) Distinguish between the following:
 (i) Null hypothesis and Alternative hypothesis
 (ii) Simple hypothesis and Composite hypothesis

(a) State the error and Type of error
 (iii) Type I error and Type II error

